Algorithms across Interference Models

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- Wireless devices located in a metric space
- Each communication request has value $v_i > 0$
- Transmissions with Interference (and Noise)

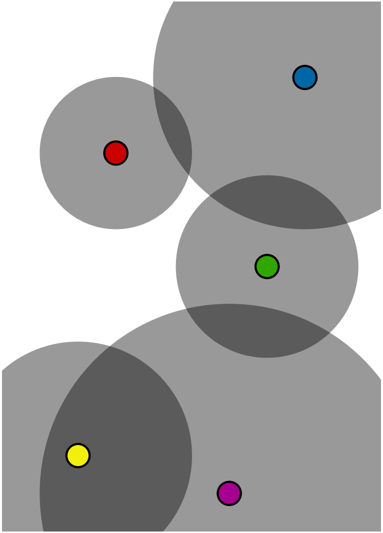
Problem: Capacity Maximization

Maximize total value of successful transmissions.

- Basic problem for illustration
- Foundational for many more complicated tasks

Design Algorithms across Interference Models

Interference?



Disk Graph Model

- Users are transmitters in the plane
- User i has a transmission range
- Two transmitters can get assigned the same channel if their ranges do not intersect.

Set I of users is successful if there is no intersection among ranges of users in I, i.e., I is an independent set in the intersection graph.

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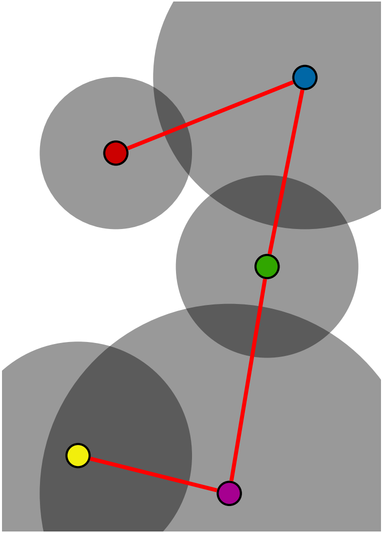

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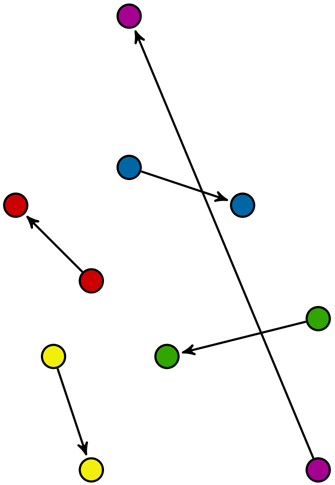

Protocol Model

Underlying Metric Space (V, d)
Requests between points in V

- d_{ij} distance between sender s_i and receiver r_j
- Success if every chosen sender is far
- Condition based on constant $\Delta > 0$:

$$d_{ij} \geq (1 + \Delta) d_{jj}$$

Successful requests are simultaneously feasible w.r.t. their distance condition.

We can build a directed conflict graph G. Set I of successful users is an independent set in G.

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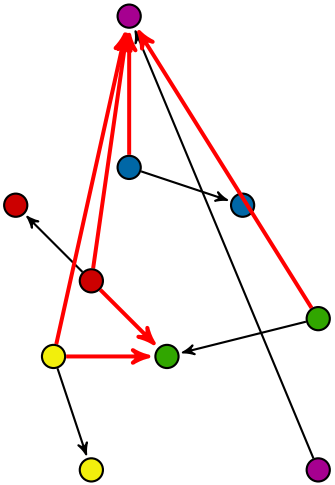

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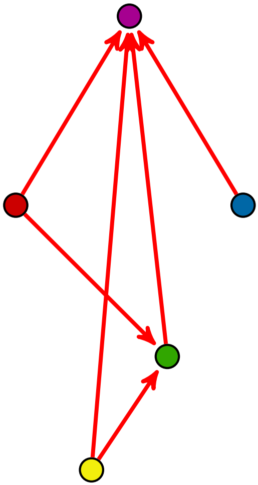

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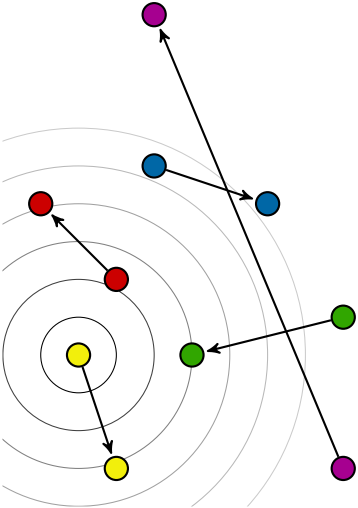

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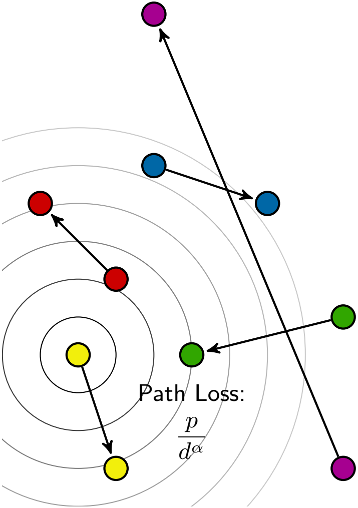

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Parameter:

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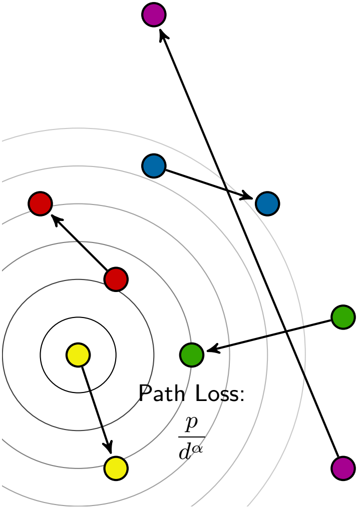


Physical Model

Underlying Metric Space (V, d)
Requests between points in V

Parameter:

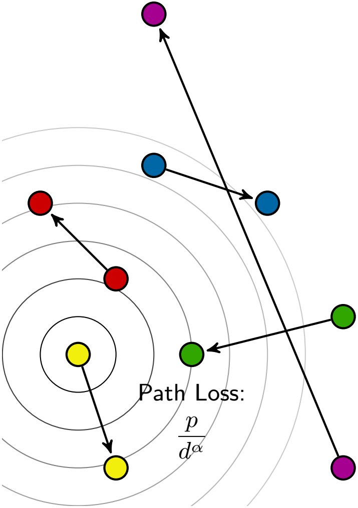
- Path loss exponent α
- Decay: $g_{ij} = 1/d_{ij}^{\alpha}$
- Threshold $\beta > 0$
- Noise $\nu \geq 0$

SINR Condition:

$$g_{ii} \cdot p_i \quad \geq \quad \beta \cdot \left(\nu + \sum_{j \neq i} g_{ji} \cdot p_j \right)$$

Successful requests are simultaneously feasible w.r.t. their SINR condition.

Physical Model of Interference



Weighted Conflict Graph

- Fixed distances d_{ij} and powers p_i
- Complete directed graph
- $w(i,j)$ for ordered pair of requests i, j
- Measures impact of interference of i on j, relative to j's signal strength
- Affectance:

$$w(i,j) = \frac{\beta \cdot g_{ij} \cdot p_i}{g_{jj} p_j - \beta \nu}$$

SINR Condition:

$$\sum_{j \neq i} w(j,i) \leq 1$$

Inductive Independence

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For request j, ordering π of requests, the backward set of j is

$$\Gamma_\pi(j) = \{i \mid \pi(i) < \pi(j)\} \ .$$

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G has inductive independence number $\rho \Leftrightarrow$ The best ordering bounds the incoming weight from every independent set in every backward set to at most ρ.

Definition

The inductive independence number of G is the minimum number ρ s.t. there is ordering π which has for all j and independent sets I:

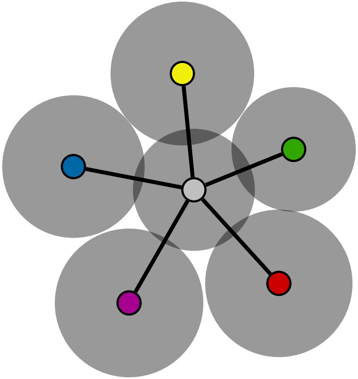
$$\sum_{i \in I \cap \Gamma_\pi(j)} \bar{w}(i,j) \leq \rho \ .$$

Proposition

For disk graphs, the inductive independence number ρ is at most 5.

Idea:

- Non-increasing order of radius
- Geometric Argument:
  At most $\rho = 5$ intersecting disks with larger radius and without mutual intersection. $\square$



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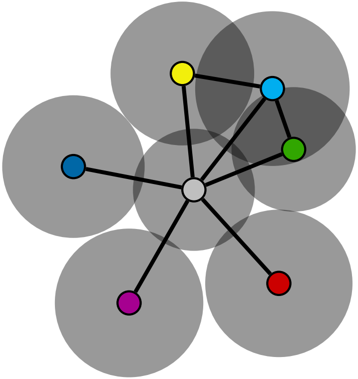

All prominent interference models have small upper bounds on ρ. These bounds hold even for trivial orderings.

Model	Order	Bound	Ref.
Disk Graphs	Radius	5	[Folklore]
Protocol Model	Length	$\left\lceil \frac{\pi}{\arcsin \frac{\Delta}{2(\Delta+1)}} \right\rceil - 1$	[Wan, MobiCom'09]
IEEE 802.11 model	Length	23	[Wan, MobiCom'09]
Distance-2-Match	Radius	$O(1)$	[Barrett et al, PERCOMW'06]
Distance-2-Color	Radius	$O(1)$	[Hoefer et al, SPAA'11]
SINR, Monotone	Length	$O(\log n)$	[Kesselheim, Vöcking, DISC'10]
SINR, Mean	Length	$O(1)/O(\log\log\Delta)$	[Halldórsson et al, SODA'13]
SINR, Power Ctrl.	Length	$O(1)$	[Kesselheim SODA'10, ESA'12]

Algorithms for Unweighted Conflict Graphs [Akcoglu et al, 2002]

Greedy Algorithm for MaxIS

- Every node has initial budget $b_i = 1$
- For each node i in reverse order of π do:
- If current $b_i > 0$ do:
- For every backward neighbor j set $b_j \to \max\{b_j - b_i, 0\}$.
- Pick all nodes with $b_i > 0$

- A local ratio argument shows that greedy computes a ρ-approximation.
- There is no $\rho/\omega(\log^4 \rho)$-approximation algorithm for independent set. Follows from a lower bound in regular graphs. [Chan, STOC'13]

Algorithms for Unweighted Conflict Graphs [Akcoglu et al, 2002]

Greedy Algorithm for (node-)weighted MaxIS

- Every node has initial budget $b_i = v_i$
- For each node i in reverse order of π do:
- If current $b_i > 0$ do:
- For every backward neighbor j set $b_j \to \max\{b_j - b_i, 0\}$.
- Initialize $S = \emptyset$.
- For each node i in order of π do:
- If $b_i > 0$ and $S \cup \{i\}$ is independent, add i to S.

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LP relaxation:

$$\begin{aligned} \max \quad & \sum_i v_i x_i \\ \text{s.t.} \quad & \sum_{\substack{i \in V \\ \pi(i) < \pi(j)}} \bar{w}(i,j) \cdot x_i \leq \rho \qquad \forall j \\ & x_i \leq 1 \qquad \forall i \end{aligned}$$

Algorithms for Edge-Weighted Conflict Graphs [H., Kesselheim, Vöcking, SPAA'11]

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- Solve the LP relaxation, solution x^*. Use randomized rounding with $x_i^*/4\rho$. Apply greedy conflict resolution in the ordering of π s.t. resulting set S fulfills stronger constraints: $\sum \bar{w}(i,j) \cdot x_i \leq 1/2$.

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- Integral solution S is a 8ρ-approximation, but it is not feasible in the forward direction.
- Partition S into $\lceil \log n \rceil$ feasible independent sets, pick the best one. This gives a $(8\rho \cdot \log n)$-approximation for the problem.

Approximation Results for Capacity Maximization [Hoefer, Kesselheim, EC'12]

We consider the problem for k channels.

- Requests on each channel form an independent set in the conflict graph.
- Request i has value $v_i(S)$ for subset $S \subseteq [k]$.
- Improved results depending on structure of the $v_i(S)$.

	Unweighted CG	Weighted CG	
1 Channel	ρ	$O(\rho \cdot \log n)$	[AADK'02, HKV'11]
k Channels	$O(\rho \cdot \sqrt{k})$	$O(\rho \cdot \sqrt{k} \cdot \log n)$	[HKV'11]
k-Multi-Unit	$O(\rho)$	$O(\rho \cdot (\log n + \log k))$	[HK'12]
k-MRS	$O(\rho)$	$O(\rho \cdot \log n)$	[HK'12]

Spectrum Auctions

Requests posted by strategic agents

- Valuations $v_i(S)$ are private information.
- Agents receive channel access and must pay for it
- Agent maximizes: Value of obtained channels minus payments.
- Allocation algorithm is public knowledge.
- Affectances are public knowledge.

Spectrum Auctions:

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Randomized Meta-Rounding yields a randomized algorithm with the same approximation bounds and truthfulness in expectation. [Lavi, Swamy JACM'11]

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Suppose requests all have value $v_i = 1$, affectances are determined in advance, and requests are revealed in uniformly random order.

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Sample-and-Inject for Unweighted Conflict Graphs

- Reject the first $k = Binom(n, 0.5)$ requests, denote this set by M_1
- Set output $S = \emptyset$
- For each subsequent request i do
- Would Greedy on $M_1 \cup i$ take i? No: Reject i.
- Reject i with probability $1 - {}^{1}/_{2\rho}$.
- If i survived and $S \cup i$ is IS, accept i and set $S \leftarrow S \cup i$.
- Otherwise reject i.

Theorem

Sample-and-Inject is $O(\rho^2)$-competitive for unweighted conflict graphs and requests with $v_i = 1$.

More generally, we obtain the following bounds:

	Unweighted CG	Weighted CG
$v_i = 1$	$O(\rho^2)$	$O(\rho^2 \log^2 n)$
arbitrary	$O(\rho^2 \log n)$	$O(\rho^2 \log^3 n)$

Extensions:

- For many other stochastic adversaries, e.g., prophet inequalities or periodic optimization with limited changes, we get the same bounds.
- Advanced scenarios with arrival and departure dates, at the expense of an additional $O(\log n)$ factor.

Extensions

At the expense of additional $O(\log n)$ factors, algorithms based on the inductive independence number allow to solve numerous more complicated tasks across interference models.

- Discrete Utility Functions for different SINR

  [Kesselheim ESA'12]

- Latency Minimization and Distributed Scheduling

  [Kesselheim, Vöcking DISC'10; Halldorsson, Mitra ICALP'11]

- Dynamic Packet Arrival and Stability

  [Kesselheim PODC'12; Asgeirsson et al SIROCCO'12]

- Wireless Connectivity

  [Halldorsson, Mitra SODA'12; PODC'12]

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- Are there other equally useful parameters for designing approximation algorithms that allow abstraction/application across interference models?
- What about distributed algorithms and restricted feedback models?