Modeling Spatial Variability of Wireless Signals

Marijke Bodlaender *Magnús M. Halldórsson*

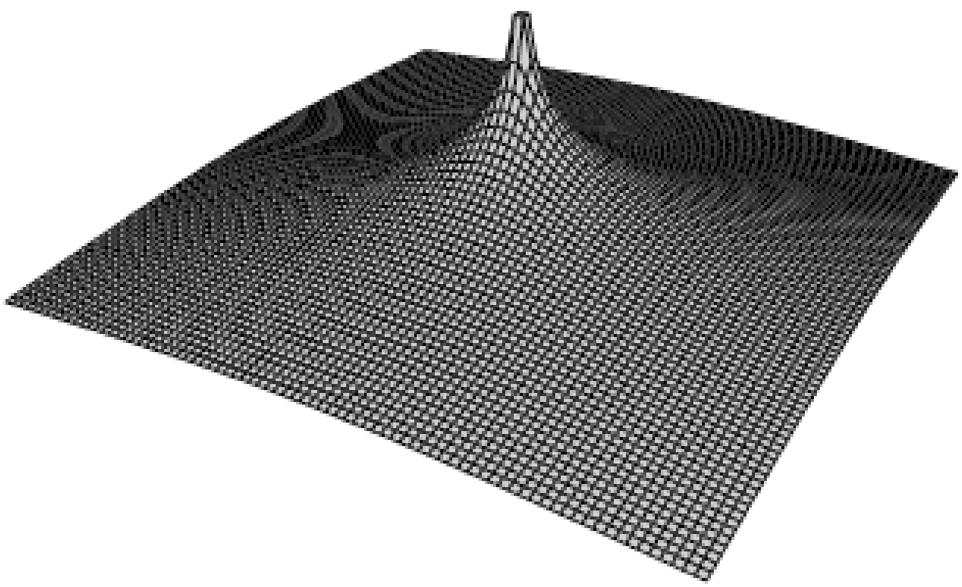
Reykjavik University, Iceland

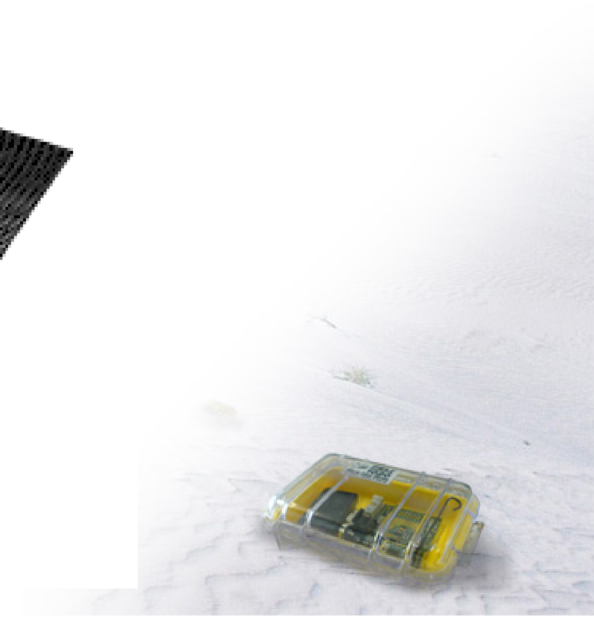
Signal transmission

- Radio signal diminishes as it travels
- In the <u>free space</u>, the *path loss* is proportional to d^{α} where

  d : distance

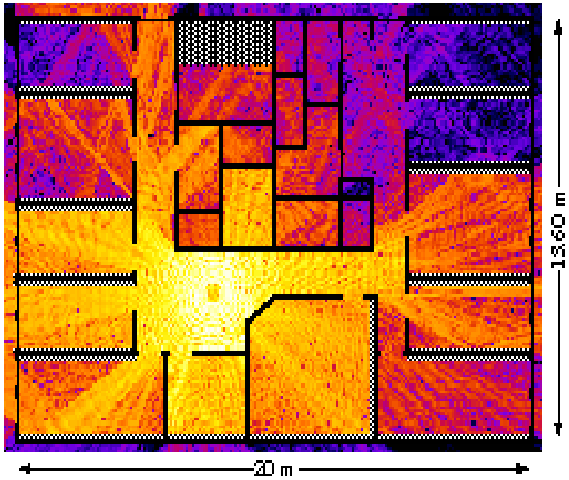
  α : path loss constant (usually, $2 < \alpha < 6$), depends on medium

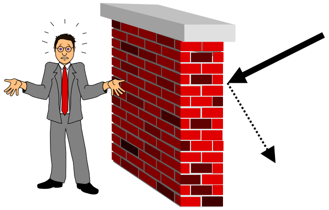




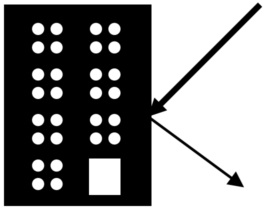
Modeling Reality

Can we capture more of the complications of reality in algorithmically usable models?

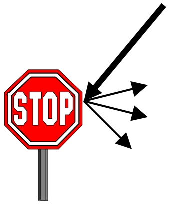




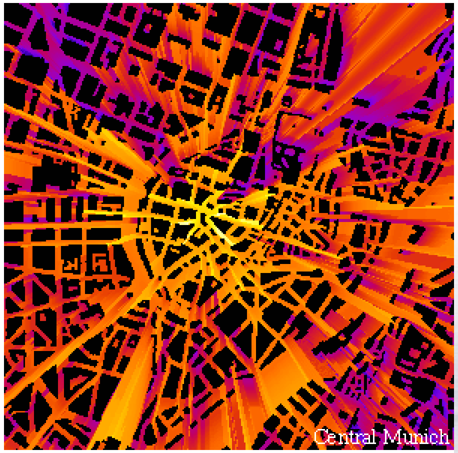
shadowing



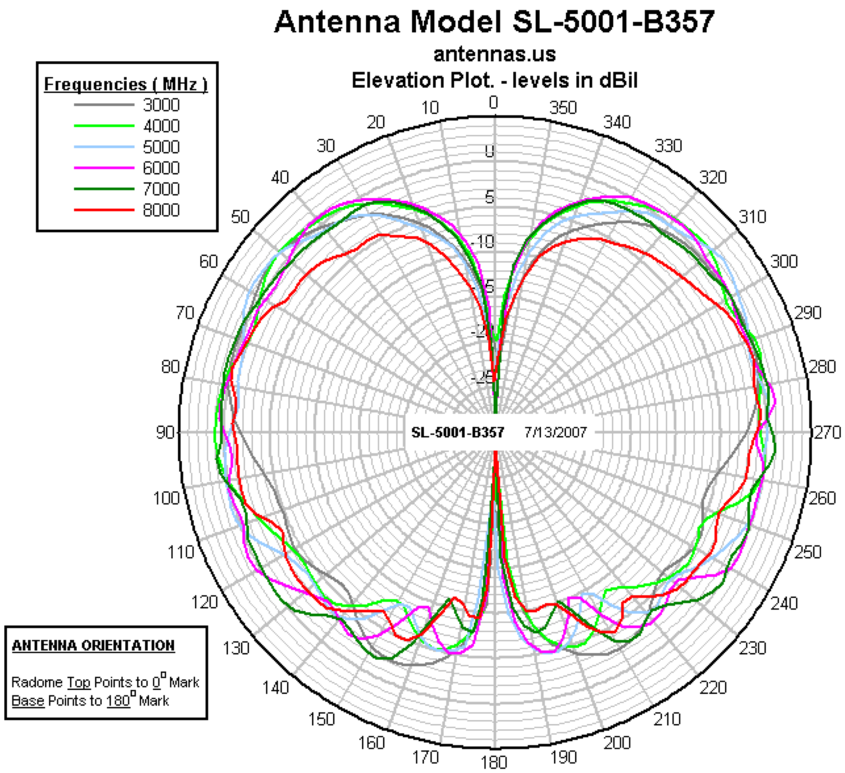
reflection



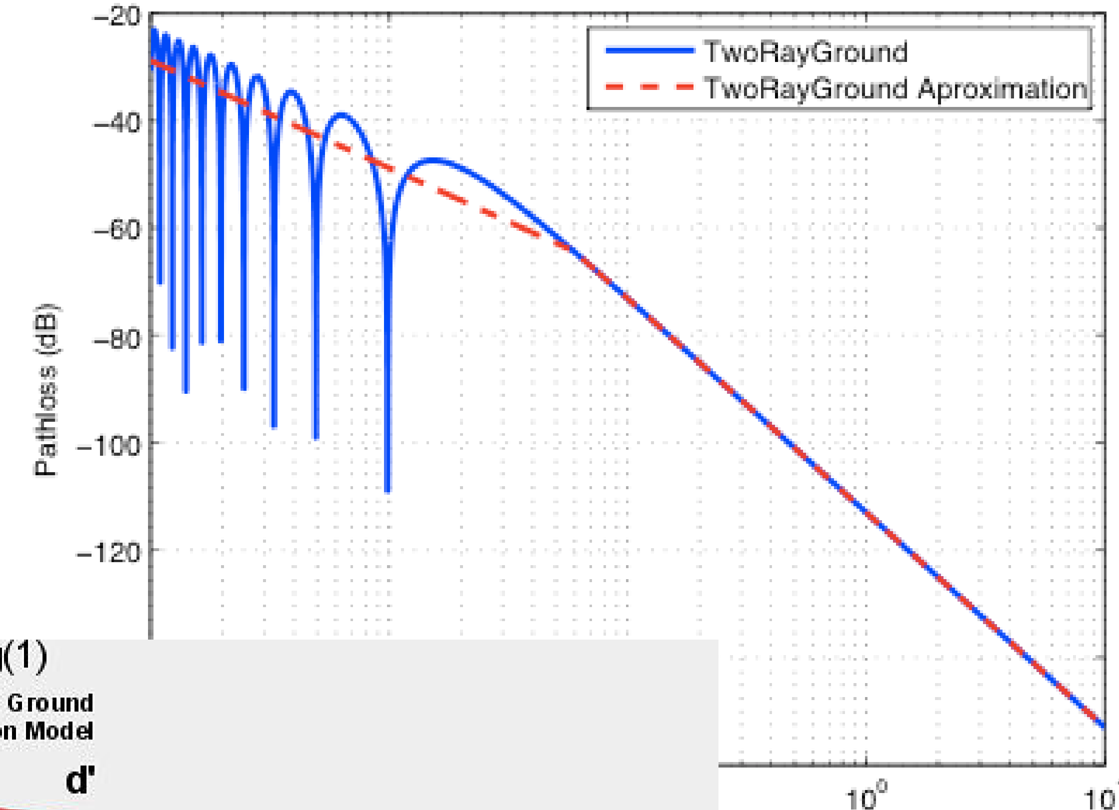
scattering

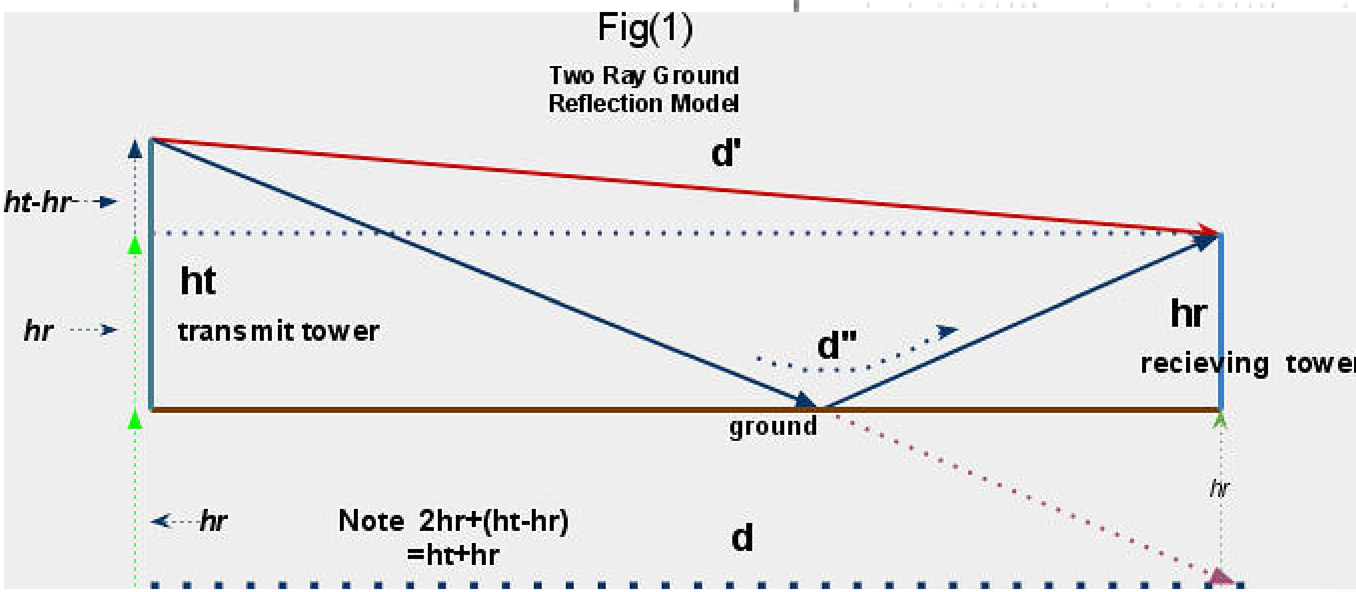


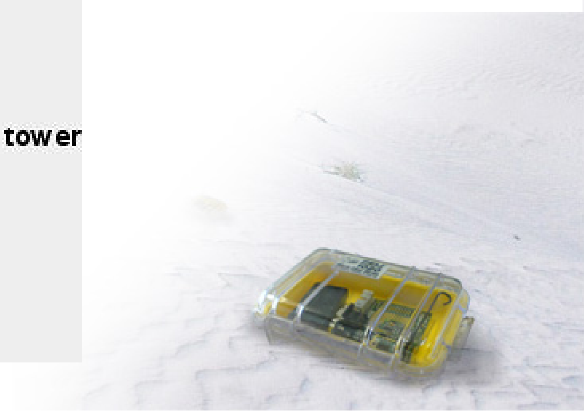
Anisotropic Antennas



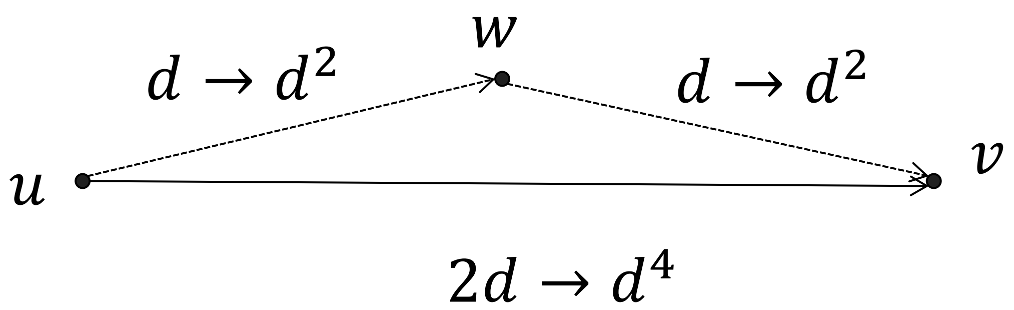
Two-ray model

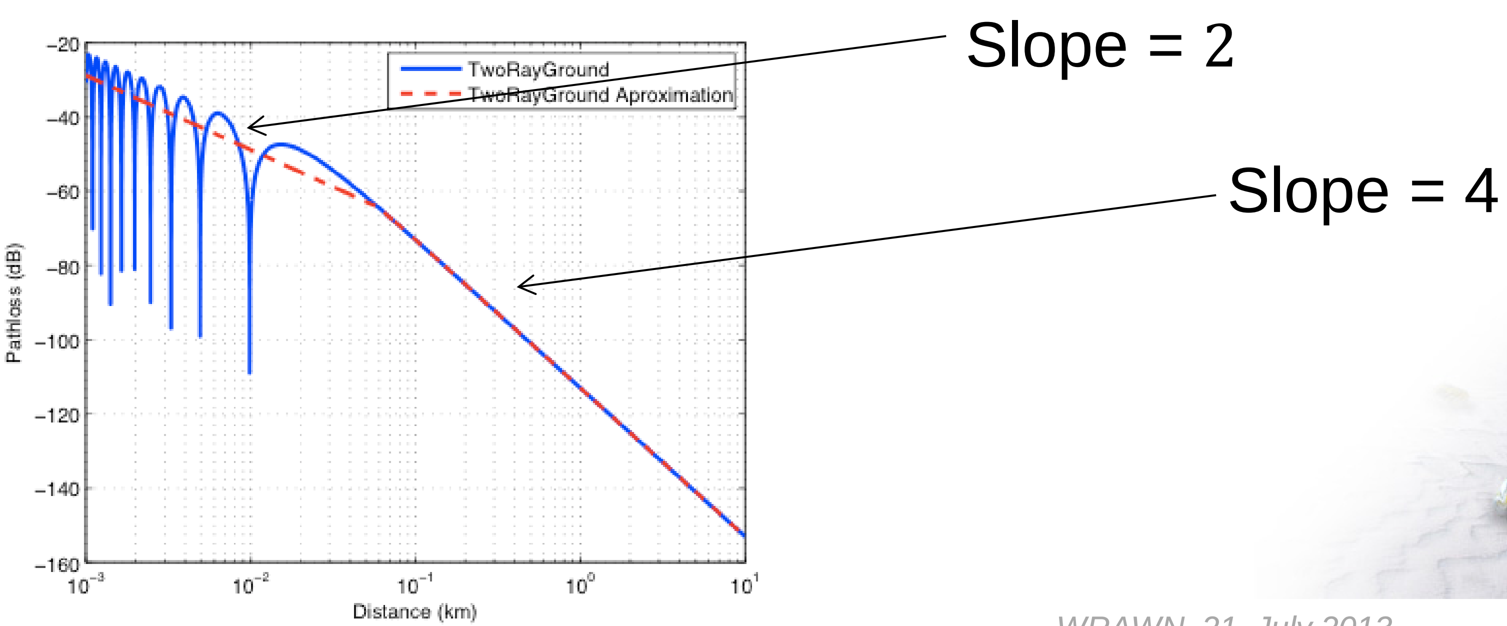






Two-ray model is not captured by a metric





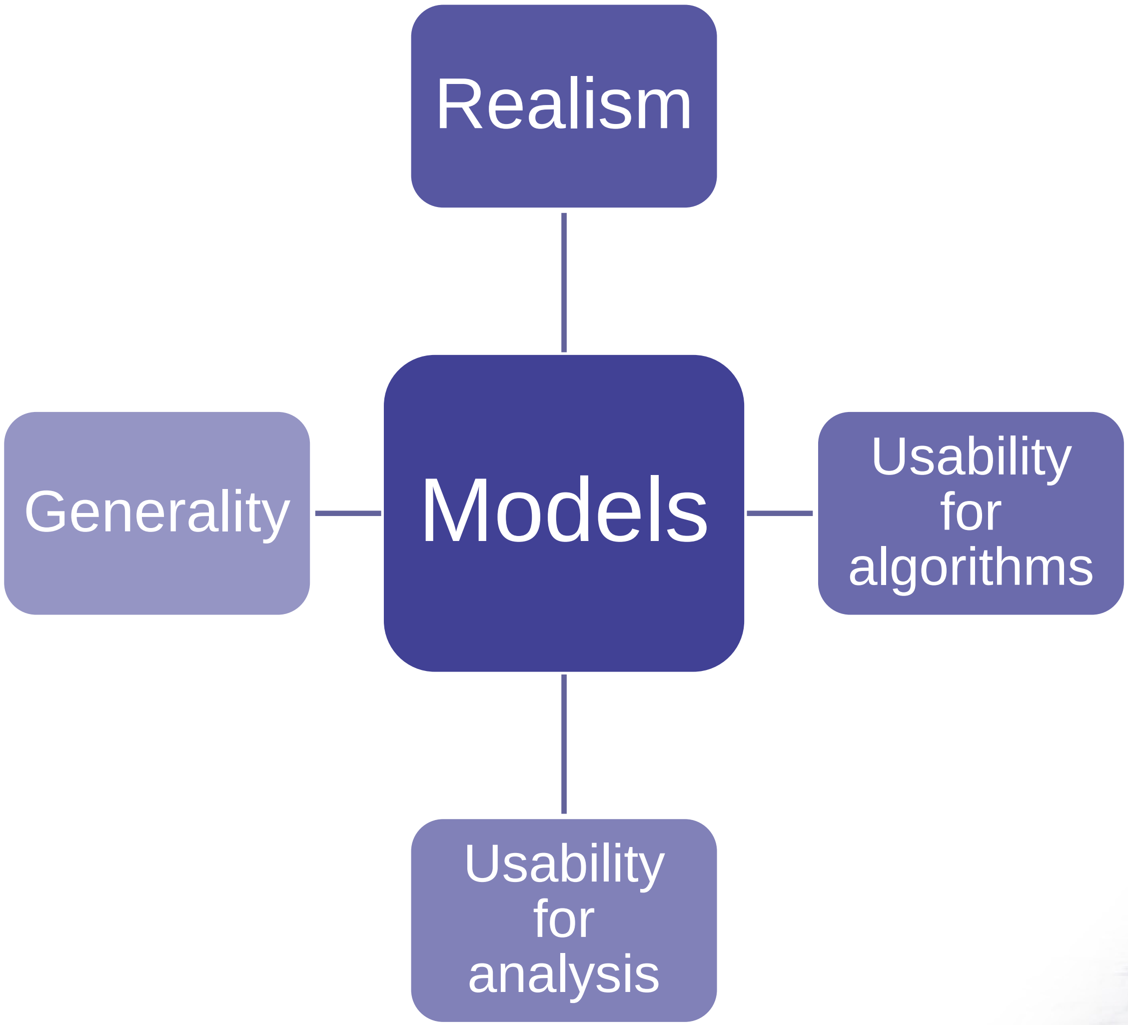
Abstract SINR

- One very general approach is to model these effects by a general (arbitrary) matrix:
  g(v,w) = decay of signal from link *v* to link *w*
- Can capture arbitrary signal propagation

$$\frac{P_v g(v,v)}{N + \sum_w P_w g(w,v)} \geq \beta$$

- Problem: Too general
  - Capacity problem becomes harder than Max Independet Set in graphs
  - Scheduling becomes harder than Graph Coloring
- → Must find a way to utilize the inherent geometry

Tradeoffs in Models



Vexing...

- One idea: Point-based density
  - Define „density“ of each point
  - As message traverses through the point, signal decays according to the density
  - Need some constraints on max density, or density difference
- Another idea: Path-based decay
  - Allows for asymmetry

- We want:
  - Simplicity
  - Generality
  - Compatibility with geometric SINR
  - Minimal effort of transferring known results

How are distances actually used in the proofs?

$$d_{ux} = d(s_u, r_x) \leq d(s_u, s_v) + d(s_v, r_x) \leq 3d(s_v, r_x) = 3d_{vx} \ ,$$

$$a_v(x) \leq c_x \frac{P_u}{d_{vx}^{\alpha}} \frac{\ell_x^{\alpha}}{P_x} \leq c_x \frac{3^{\alpha} P_u}{d_{ux}^{\alpha}} \frac{\ell_x^{\alpha}}{P_x} = 3^{\alpha} a_u(x) \ ,$$

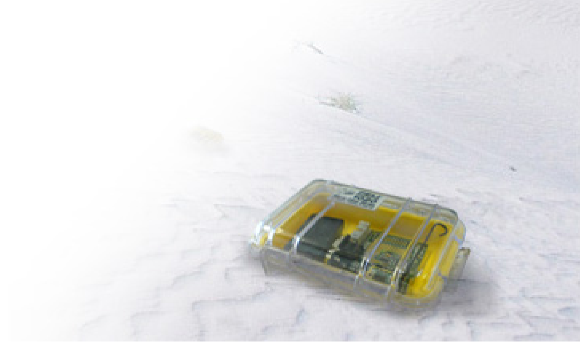
Proposal: Decay Model

Geometric SINR model

$$\frac{P_v / {d_{vv}}^{\alpha}}{N + \sum_{u \in S} P_u / {d_{uv}}^{\alpha}} \geq \beta$$

Decay model

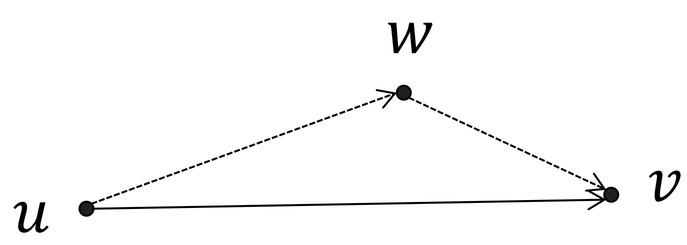
$$\frac{P_v / f_{vv}}{N + \sum_{u \in S} P_u / f_{uv}} \geq \beta$$



ϕ-inframetric

- Decay function is a *quasi-inframetric*:

$$f_{uv} \leq \varphi \max(f_{uw}, f_{wv})$$



- In geometric SINR, $\varphi = 2^{\alpha}$

Algorithms for Capacity Maximization

Input: Set *L of links*;
Output: Feasible subset X;

for each link l_w in *L* in increasing order of ~~length~~ decay **do**
  if $a_S(w) + a_w(S) \leq \frac{1}{4}$
    then add l_w to S
Output $X = \{l_w \in S : a_S(w) \leq 1\}$

Old vs. New Proofs

- Old form:

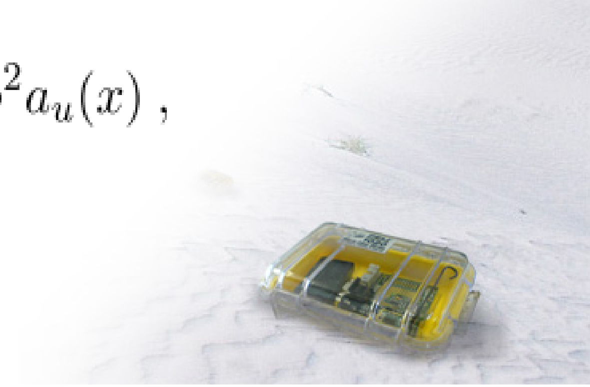
$$d_{ux} = d(s_u, r_x) \leq d(s_u, s_v) + d(s_v, r_x) \leq 3d(s_v, r_x) = 3d_{vx} \ ,$$

$$a_v(x) \leq c_x \frac{P_u}{d_{vx}^{\alpha}} \frac{\ell_x^{\alpha}}{P_x} \leq c_x \frac{3^{\alpha} P_u}{d_{ux}^{\alpha}} \frac{\ell_x^{\alpha}}{P_x} = 3^{\alpha} a_u(x) \ ,$$

- New form:

$$f_{ux} \leq \phi \max(f(s_u, s_v) + f(s_v, r_x)) \leq \phi^2 f_{vx} \ ,$$

$$a_v(x) = c_x \frac{P_v}{f_{vx}} \frac{f_x}{P_x} \leq c_x \frac{\phi^2 P_u}{f_{ux}} \frac{f_x}{P_x} = \phi^2 a_u(x) \ ,$$

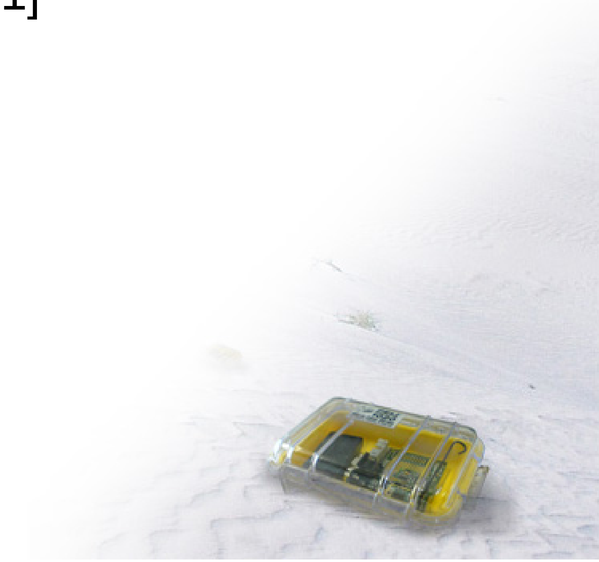


Results

- Oblivious-power Capacity is $O(\varphi^6)$-approximable [H, Mitra,‘11]
- Same for Arbitrary-power Capacity [Kesselheim, SODA‘11]
- Results carry over to the Connectivity problem

- Key structural lemmas implies results for:
  - Distributed scheduling [KV10, HM‘11]
  - Admission control / spectrum sharing [HM ‘12]
  - Weighted capacity with linear power [HM‘12]
  - Distributed capacity via regret minimization [AM‘11]

- Changes to proofs minimal
  - Validates research on the basic SINR model!



Open Issues

- Probabilistic variability
  - Results can be combined with Rayleigh fading [Dams et al ‘12], for same performance ratio (H, unpub)
  - „Shadowing“ is yet to be studied
- Temporal variability
  - Major issue, largely untouched
  - How is it dependent across time? What time window is static?
- Adversarial variability

- How bad can φ get?
  - Experimental validation needed.

Thank you

Come to Iceland, if you need to cool down in summer!